

INFLUENCE OF THE  
MAGNETIC AMPLIFIER ON  
THE VOLTAGE  
REGULATION PROBLEM

BY  
W. R. WAKELAND

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-

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ON THE VOLTAGE REGULATION PROBLEM

by

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Lieutenant, United States Navy

Submitted in partial fulfillment  
of the requirements  
for the degree of  
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## PREFACE

Since the magnetic amplifier has increased in use in the recent years, an investigation of a voltage regulation circuit using the magnetic amplifier was performed at the United States Naval Postgraduate School, Annapolis, Maryland, during the period January to June, 1951. This work was performed with the timely guidance of Professors W. C. Smith and R. C. H. Wheeler, of the Electrical Engineering Department.



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## CHAPTER I

### GENERAL REGULATION PROBLEM

#### 1. Introduction

In the recent years the magnetic amplifier has grown in popularity for use in control circuits. The advantage of no electronic or movable parts is the principal reason why this saturable reactor phenomenon has been explored. This thesis will develop the general regulation problem and discuss the important points in order to bring these considerations to the fore. It will discuss the magnetic amplifier characteristics which seem to be involved in the regulation problem. It will propose a design schedule for a regulation system utilizing a magnetic amplifier. This design schedule assumes sharp and complete saturation of the core material. It also is based on "garden variety" circuits, amplification varying directly with time constant, long-transient theory, and on information available to compute "power output vs volume of core" curves.

The transient performance of a generator voltage regulating system is the terminal voltage, as a function of time, following a voltage disturbance or a change in load. For linear systems the parameters of the general solution are the time constants of the alternator, exciters, damping transformers, and over-all voltage amplification. It is pertinent at this time to develop analytically and logically the basic facts to be considered in the design of voltage regulating systems.



In this treatment the alternator voltage regulating system will be taken to include the alternator, the exciters, the regulator (or voltage sensing device) and any other elements entering into the mathematical formulation of the regulating problem.

## 2. Two time delay system.

First a simple "two time delay" system will be investigated to point out the method of analysis, and certain fundamental conclusions which can be drawn from this simple system. This system (Figure 1) consists of an alternator, excitor and a regulator, without time delay, of amplification  $M$ . Only the transient part of the envelope of the terminal voltage will be considered, since this would be superimposed upon the envelope existing before a voltage disturbance. This is likewise true of all other voltages and currents. In considering the problem thusly we are assuming straight line saturation, and initial conditions can be considered zero.



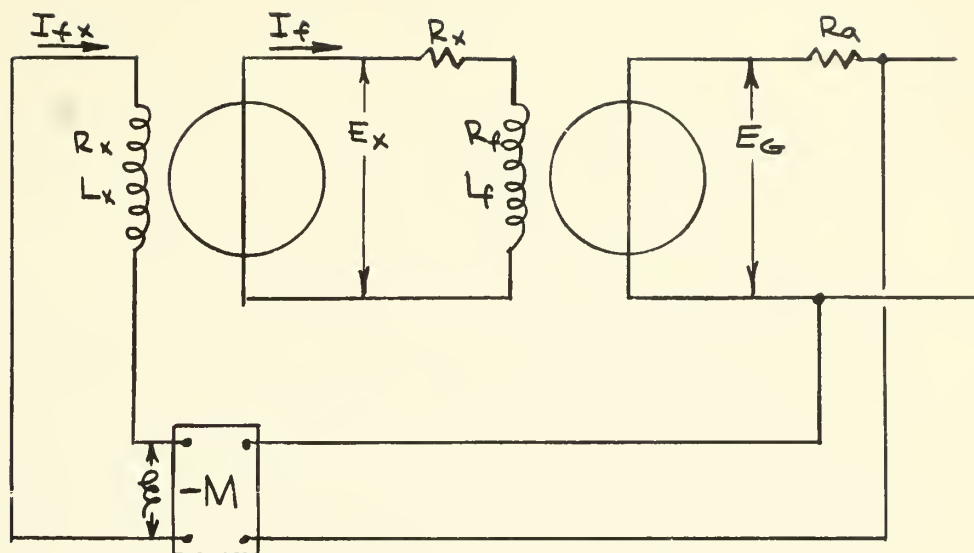


FIGURE 1  
TWO TIME DELAY SYSTEM

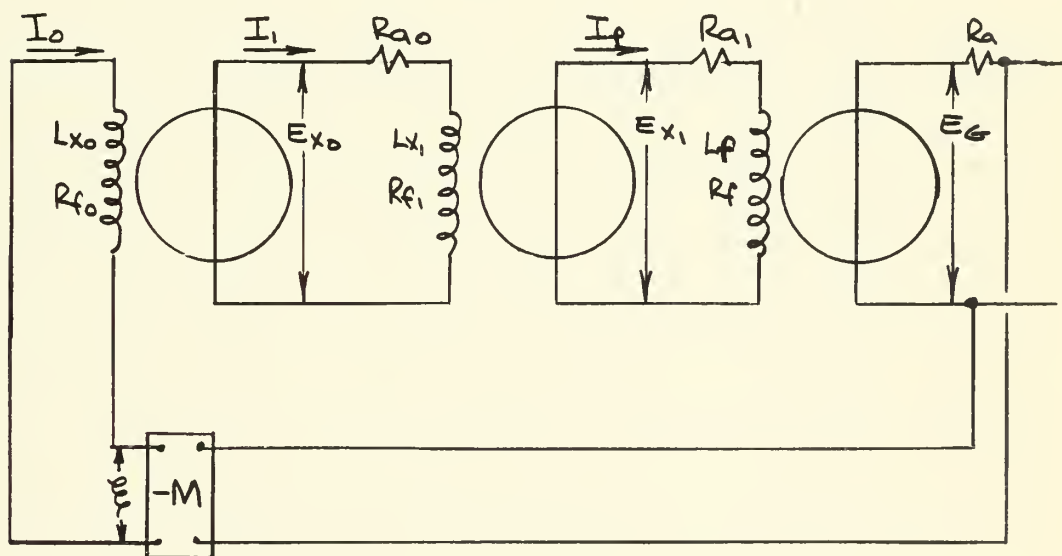


FIGURE 2  
THREE TIME DELAY SYSTEM





$$E_G = K'_G \phi_G S_G = K_G \phi_G \quad (\text{constant speed})$$

where:  $\phi_G = \text{flux}$  ;  $K_G = \text{proportionality constant}$ .

$$\text{also } \phi_G = N_1 I_f$$

$$E_x = I_f (R_x + R_f) + L_f \frac{dI_f}{dt}$$

$$\text{also } E_x = K'_x \phi_x S_x = K_x \phi_x \quad (\text{constant speed})$$

where:  $\phi_x = \text{flux of exciter}$  ;  $K_x = \text{exciter constant}$ .

$$\text{also } \phi_x = N_2 I_{fx}$$

$$-M \mathcal{E} = \left( R_{fx} + L_x \frac{d}{dt} \right) I_{fx}$$

where:  $\mathcal{E} = E_G - V u(t)$ , and  $V u(t)$  is voltage disturbance.

using Laplace transforms:

$$\bar{I}_{fx} = \frac{-M \bar{\mathcal{E}}}{R_{fx} + L_x S} \quad ; \text{ bar over symbol indicates}$$

Laplace transform, "S" is Laplacian variable.

$$\bar{E}_x = \frac{-K_x N_2 M \bar{\mathcal{E}}}{R_{fx} + L_x S} = \bar{I}_f (R_1 + L_f S)$$

$$\bar{I}_f = \frac{-K_x N_2 M \bar{\mathcal{E}}}{(R_{fx} + L_x S)(R_1 + L_f S)} = \frac{-K_x N_2 M \bar{\mathcal{E}}}{(T_0 S + 1)(T_2 S + 1) R_{fx} R_1}$$

$$\text{where: } \frac{L_x}{R_{fx}} = T_0 ; \frac{L_{fx}}{R_1} = T_2 ; R_1 = R_f + R_x$$



$$\bar{\Phi}_G = N_1 \bar{I}_f = \frac{-K_x N_2 N_1 M \bar{E}_G}{(T_0 S + 1)(T_2 S + 1) R_{fx} R_1}$$

$$\bar{E}_G = K_G \bar{\Phi}_G \quad \therefore \text{let } C = \frac{K_G K_x N_1 N_2 M}{R_{fx} R_1}$$

$$\text{then: } \bar{E}_G = \frac{-C(\bar{E}_G - \sqrt{u(t)})}{(T_0 S + 1)(T_2 S + 1)}$$

$$\bar{E}_G = \frac{C V}{S[(T_0 S + 1)(T_2 S + 1) + C]}$$

roots of denominator are:

$$S = \frac{-(T_0 + T_2) \pm \sqrt{(T_0 + T_2)^2 - 4(1 + C)(T_2 T_0)}}{2 T_0 T_2}$$

Since the roots of S cannot be positive, the system is always stable for any value of C,  $T_0$ , or  $T_2$ .

The solution of the above Laplace equation is as follows:

$$E_G = \frac{C}{1 + C} V - \text{exponentially decaying oscillations or transients.}$$

$$\text{Decay time constant} = \frac{1}{2} \left[ \frac{1}{T_0} + \frac{1}{T_2} \right]$$



### 3. Three time delay system.

The three time delay system, (Figure 2), is a more general system in as much as it can be unstable. The three time delay system is by no means the most complicated, in fact, it is the simplest that can be investigated and yet be considered general. The three time delay system will first be developed without damping transformers in order to point out later the effect of damping transformers.



$$-M\bar{E} = \bar{I}_0(R_{f0} + L_0 S)$$

$$\bar{I}_0 = \frac{-M\bar{E}}{R_{f0}(T_0 S + 1)} \quad ; \quad T_0 = \frac{L_{x0}}{R_{f0}}$$

$$\bar{E}_{x1} = \bar{I}_1(T_1 S + 1)R_1 = K_1 \bar{I}_0 \quad ; \quad R_1 = R_{a0} + R_{f1}$$

$$\bar{I}_1 = \frac{-K_1 M \bar{E}}{R_{f0} R_1 (T_0 S + 1)(T_1 S + 1)} \quad ; \quad T_1 = \frac{L_{x1}}{R_1}$$

$$\bar{E}_{x2} = \bar{I}_2(T_2 S + 1)R_2 = K_2 \bar{I}_1$$

$$T_2 = \frac{L_{x1}}{R_1} \quad ; \quad R_2 = R_f + R_{a1}$$

$$\bar{I}_2 = \frac{-K_2 K_1 M \bar{E}}{R_{f0} R_1 R_2 (T_0 S + 1)(T_1 S + 1)(T_2 S + 1)}$$

$$\bar{E}_G = K_G \bar{I}_2$$

$$\frac{\bar{E}_G}{\bar{E}} = \frac{-K_G K_2 K_1 M}{R_{f0} R_1 R_2 (T_0 S + 1)(T_1 S + 1)(T_2 S + 1)}$$

$$\text{let } C = \frac{K_G K_2 K_1 M}{R_{f0} R_1 R_2}$$

$$\text{then: } \bar{E}_G = \frac{-C \sqrt{u(t)}}{(T_0 S + 1)(T_1 S + 1)(T_2 S + 1) + C}$$

For a step voltage change, and assuming stability (roots of the denominator all having negative real terms):

$$\bar{E}_G = \frac{C \sqrt{V}}{1 + C} + \text{exponentially decaying oscillations}$$





Examination of the denominator of the transfer function (a cubic) will show that it is quite possible for one of the roots to have a positive real part and therefore for the system to be unstable. In the analysis of a regulating system prior to laboratory mock up, this stability must be investigated. Two of the several methods used are "Routh's Criteria", and Evans' (1) "Root Locus" method. All methods used to solve the cubic, or determine the character of the roots of the cubic are time consuming and require computations or drawings, (Evans' "Root Locus"). However, by the use of curves published by E. L. Harder (3) in his thesis and paper which was presented at the AIEE North Eastern District meeting April 23, 1947, the over-all time constant (and hence stability) and frequency of oscillations can be determined. A more complete discussion of the use of these curves will be presented later in the paper.

In order to illustrate how a damping transformer is introduced into the regulating system and its effect, a damping transformer has been added to the three delay system in Figure 3.



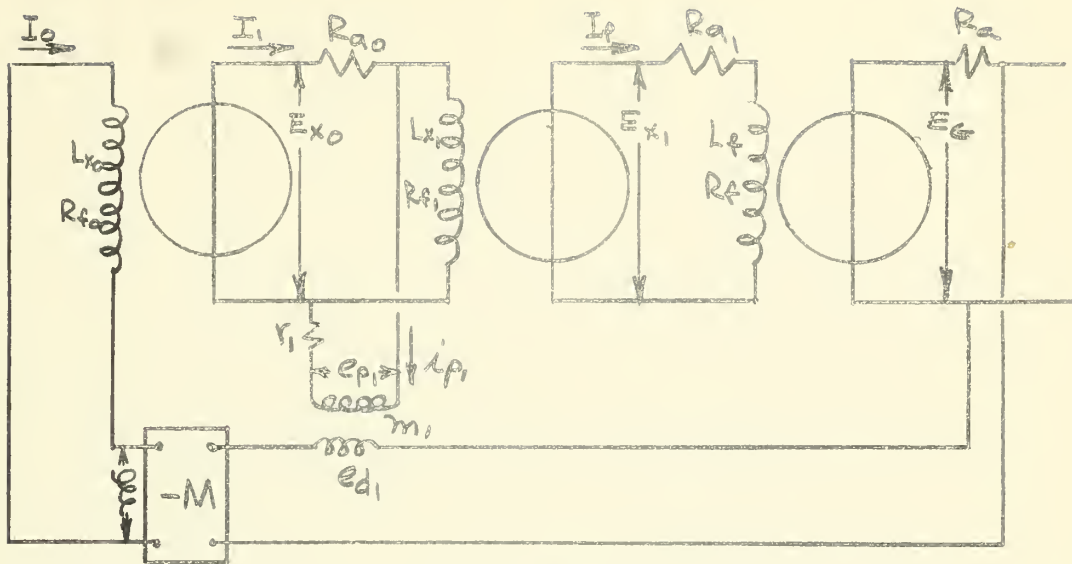


FIGURE 3  
THREE TIME DELAY SYSTEM  
WITH DAMPING TRANSFORMER

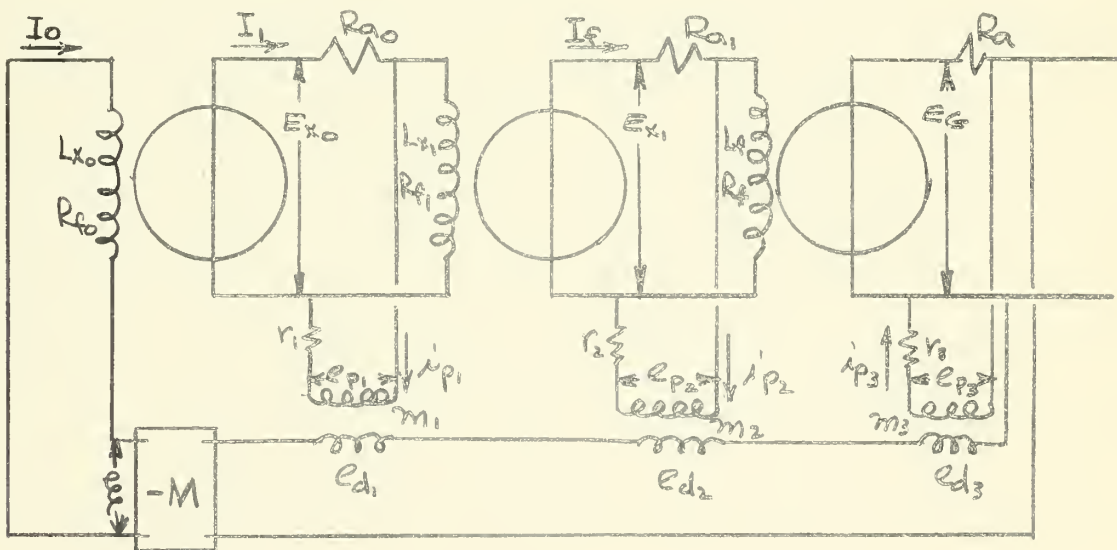


FIGURE 4  
GENERAL SYSTEM



$$-M\bar{E} = \bar{I}_o(R_{fo} + L_{xo}S)$$

$$\bar{I}_o = -\frac{M\bar{E}}{R_{fo}(T_oS+1)} \quad ; \quad T_o = \frac{L_{xo}}{R_{fo}}$$

By neglecting primary inductance for  $t_1$  :

$$i_{p1} = \frac{e_{p1}}{r_1} \quad ; \quad \frac{di_{p1}}{dt} = \frac{1}{r_1} \frac{de_{p1}}{dt}$$

$$\bar{e}_{d1} = m \frac{di_{p1}}{dt} = \frac{m_1}{r_1} s \bar{e}_{p1} = \frac{m_1 s}{r_1} \bar{E}_{x0}$$

and  $\bar{e}_{d1} = st_1 \bar{E}_{x0}$  , where  $m_1$  = mutual inductance

and  $t_1 = \frac{m_1}{r_1}$  ,  $I_1 R_{a0}$  being neglected.

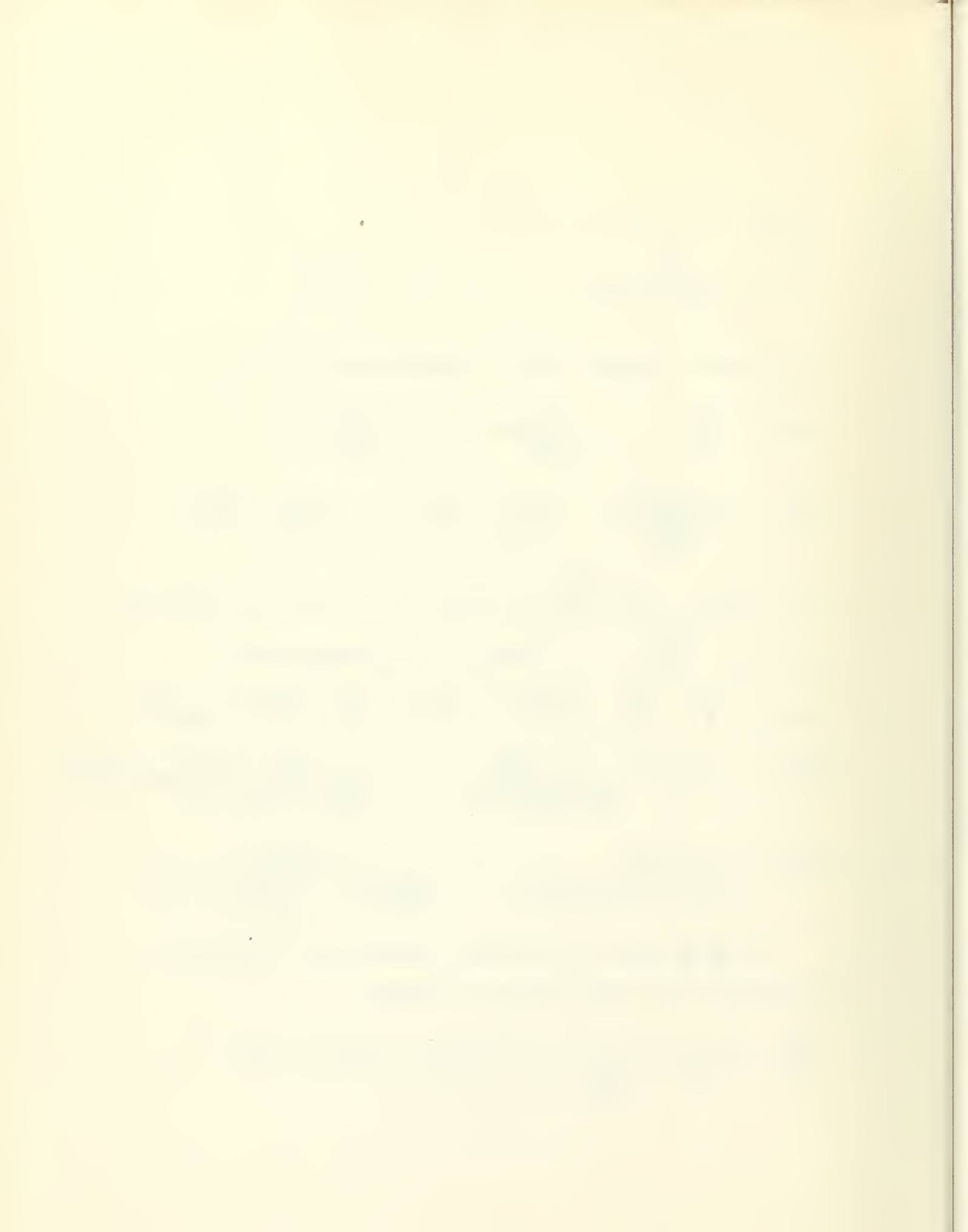
$$\text{then : } \bar{E} = \bar{E}_G - \sqrt{u(t)} + \bar{e}_{d1} = \bar{E}_G - \sqrt{u(t)} + t_1 s \bar{E}_{x0}$$

$$\bar{E}_{x0} = K_1 \bar{I}_o = \frac{-K_1 M \bar{E}}{R_{fo}(T_oS+1)} = \frac{-K_1 M (\bar{E}_G - \sqrt{u(t)} + t_1 s \bar{E}_{x0})}{R_{fo}(T_oS+1)}$$

$$\bar{E}_{x0} = \frac{-K_1 M \bar{E}_1}{R_{fo}(T_oS+1) + K_1 M t_1 s} = \frac{-K_1 M \bar{E}_1}{R_{fo} \left[ \left( T_o + \frac{K_1 M t_1}{R_{fo}} \right) s + 1 \right]}$$

If  $\bar{A} = \bar{E} - \bar{e}_{d1}$  , by following the procedure previously illustrated, the final relation becomes:

$$\frac{\bar{E}_G}{\bar{E}_1} = \frac{-C}{\left[ \left( T_o + \frac{K_1 M}{R_{fo}} t_1 \right) s + 1 \right] (T_1 s + 1) (T_2 s + 1)}$$



It will be noted here that the proper choice of  $t_1$  such that  $t_1 = -t_0 R_{f0} / K_1 M$  will reduce the system to a two time delay system which is inherently stable.

The "general" system of three time delays and three damping transformers, Figure 4, will now be developed analytically. From this solution many important curves can be taken to show the effects of various time constants.

$$\overline{e}_{d3} = m \frac{dip_3}{dt} = \frac{m_3}{r_3} s \overline{E}_G = t_3 s \overline{E}_G$$

similarly :

$$\overline{e}_{d2} = t_2 s \overline{E}_{x1}$$

$$\overline{e}_{d1} = t_1 s \overline{E}_{x0}$$

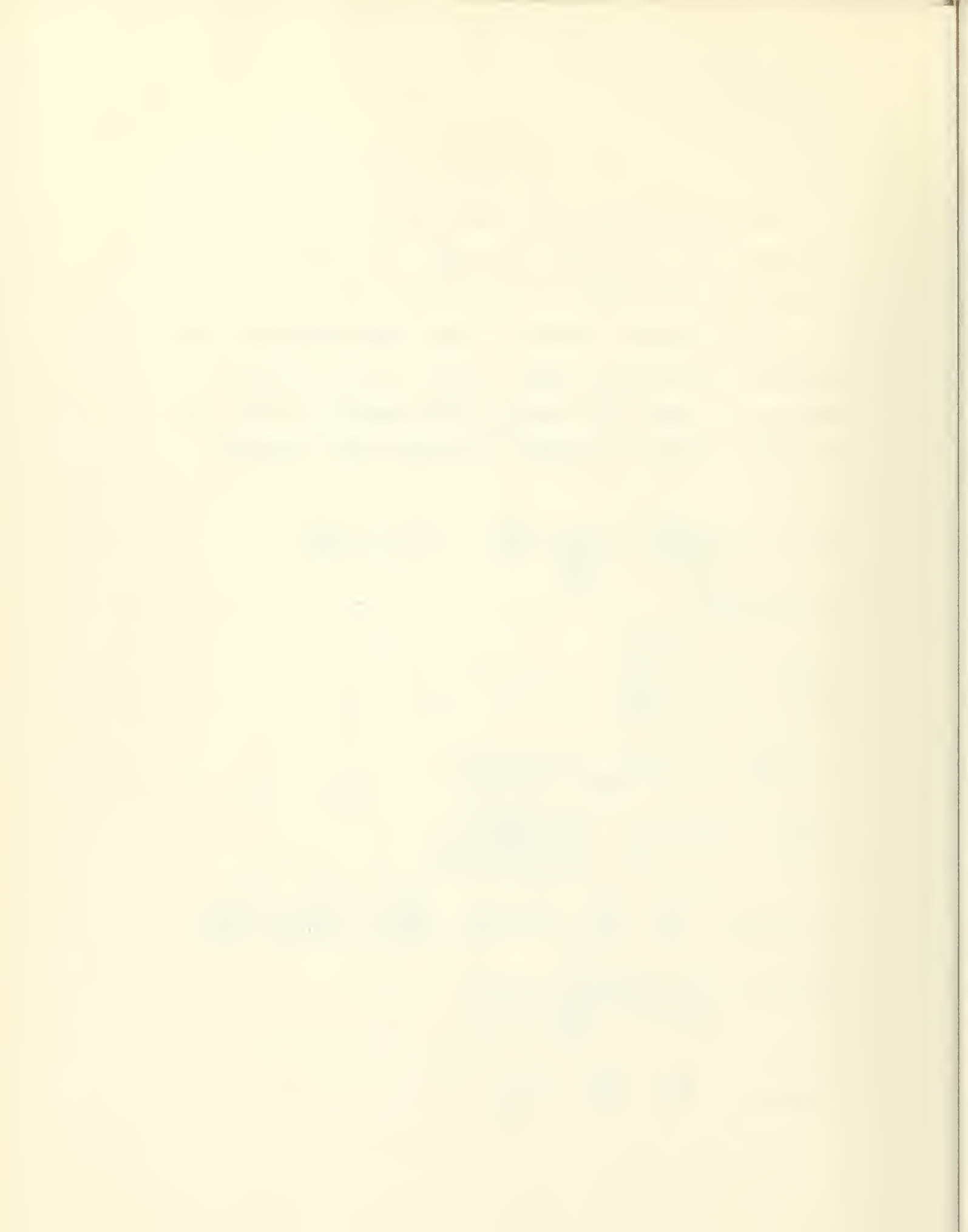
$$-M \overline{E} = I_0 (R_{f0} + L_{x0} s)$$

$$\overline{E}_{x0} = K_1 \overline{I}_0 = \frac{-K_1 M \overline{E}}{R_{f0} (T_0 s + 1)}$$

$$\text{where } \overline{E} = \overline{E}_G - \overline{V}_u(t) + \overline{e}_{d1} + \overline{e}_{d2} + \overline{e}_{d3}$$

$$\overline{E}_{x0} = \frac{-K_1 M \overline{E}_1}{R_{f0} (T_0 + \frac{M K_1}{R_{f0}} t_1) s + 1}$$

$$\text{where } \overline{E}_1 = \overline{E} - \overline{e}_{d1}$$





$$\bar{I}_1 = \frac{\bar{E}_1}{R_1 (T_1 s + 1)}$$

$$\bar{E}_{x1} = K_2 \bar{I}_1 = \frac{-K_2 K_1 M \bar{E}_1}{R_1 R_{f0} \left[ \left( T_0 + \frac{K_1 M}{R_{f0}} t_1 \right) s + 1 \right] (T_1 s + 1)}$$

$$\bar{E}_{x1} = \frac{-K_2 K_1 M \bar{E}_2}{R_{f0} R_1 \left[ \left( T_0 + \frac{K_1 M}{R_{f0}} t_1 \right) s + 1 \right] (T_1 s + 1) + K_2 K_1 M s t_2}$$

where  $\bar{E}_2 = \bar{E}_1 - \bar{E}_{d2}$

$$\bar{I}_2 = \frac{\bar{E}_{x1}}{R_2 (T_2 s + 1)}$$

$$\bar{E}_G = \frac{-K_G K_2 K_1 M \bar{E}_2}{\left\{ \left[ \left( T_0 + \frac{K_1 M}{R_{f0}} t_1 \right) s + 1 \right] [R_1 R_2] [T_1 s + 1] + K_1 K_2 M s t_2 \right\} R_2 (T_2 s + 1)}$$

$$\bar{E}_G = \frac{K_G K_2 K_1 M \overline{V u(t)}}{\left[ \left( T_0 + \frac{K_1 M t_1}{R_{f0}} \right) s + 1 \right] R_{f0} R_1 R_2 (T_1 s + 1) (T_2 s + 1) + R_2 (T_2 s + 1) (K_2 K_1 M s t_2) + (K_G K_2 K_1 M) (t_3 s + 1)}$$



It is apparent that the complexity of and the number of parameters (5) in the solution make it easy to imagine the advantage of an electrical analogy to "solve" general systems for their transient characteristics. Harder (3), as previously mentioned, has done just that in his thesis work and AIEE paper on the subject. Each curve set applies to particular values of  $A$ ,  $(T_0 + At_1)$  and  $t_3$ . In each set curves are drawn for two or more values of  $T_1$  between 0 and 1.0 as needed for interpolation. Each curve covers a range of  $t_2$  as abscissa from 0 to 1.0. The ordinates are over-all damping time constant  $T_1$  and frequency of oscillations. All parameters are per unit of  $T_2$ , the alternator time constant. This use of a per unit system affords a reduction of one parameter while grouping  $(T_0 + At_1)$  together reduces still one more the number of independent parameters. Frequency is expressed as cycles per  $T_2$ . Harder uses  $A$  for his over-all voltage amplification and lumps it into one component. His idealized system is shown in Figure 5.



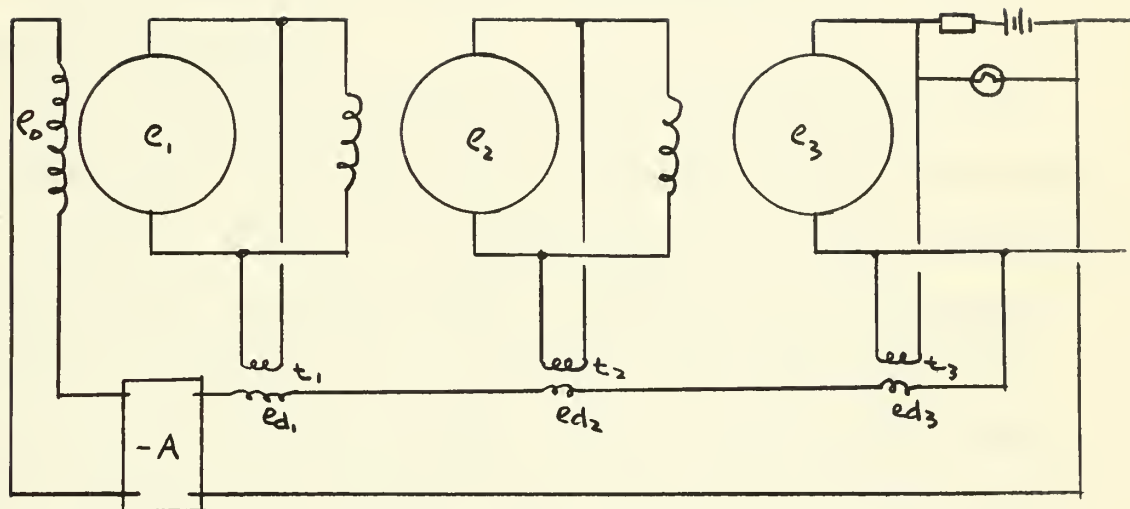


FIGURE 5  
IDEALIZED SYSTEM



Inspection of the solution for  $\bar{E}_G$  will show that the constants  $K_G, K_1, K_2, R_0, R_1, R_2$  do not appear as a group and hence such simplification cannot be accomplished in handling a practical system. However, the over-all amplification factor can be computed and factored out thereby changing, or "weighting" the time constants somewhat. This done for any system will prepare the parameters for use in Harder's handy charts.

After examining the regulator system analytically it becomes apparent what part is played by the various components of a system. A study of the curves by Harder will illustrate effects and the general solution for  $\bar{E}_G$  will give numerical results if so desired. For instance, from Figure 6, Curve set 1, and the solution for  $\bar{E}_G$ , page 5, the two time delay system is inherently stable. From the solution for  $\bar{E}_G$  page 9 and also Figure 6 curve sets 3 and 4, it is seen that reducing the first stage time delay gives a marked improvement in the system operation. Reversing the damping transformer of the first stage reduces  $(T_1 + At_1)$  and thereby improves the response of the system. The generator voltage damping transformer has a pronounced effect and should be used to achieve best performance; compare figures 6 and 7. Many other characteristics can be deduced from a comprehensive study of these sets of curves.





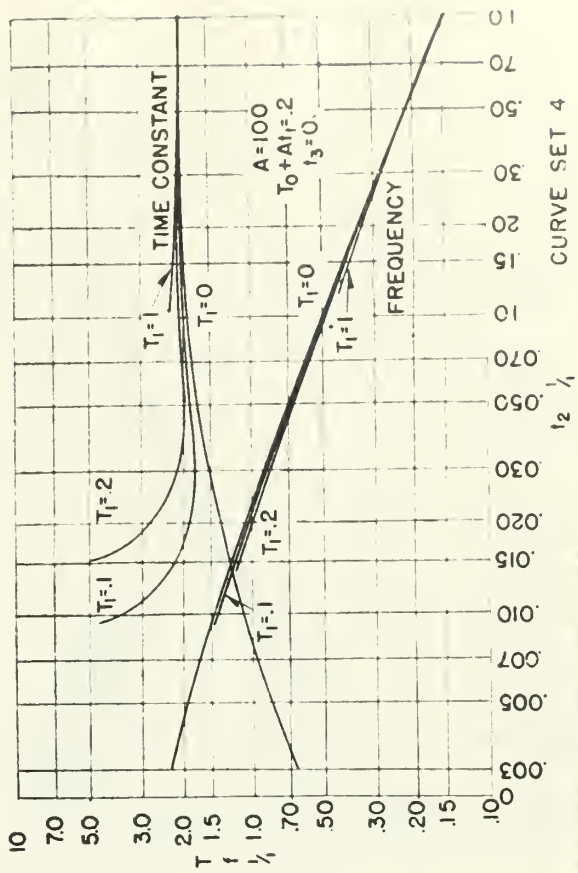
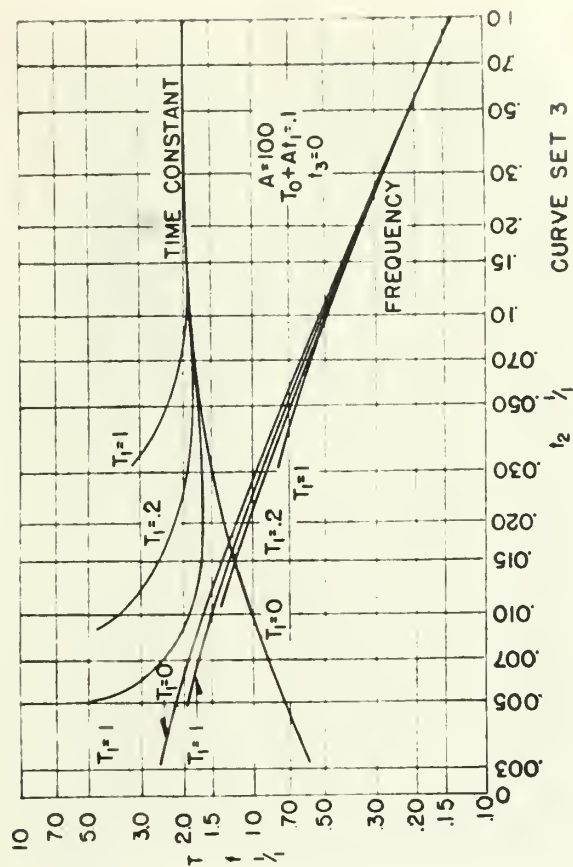
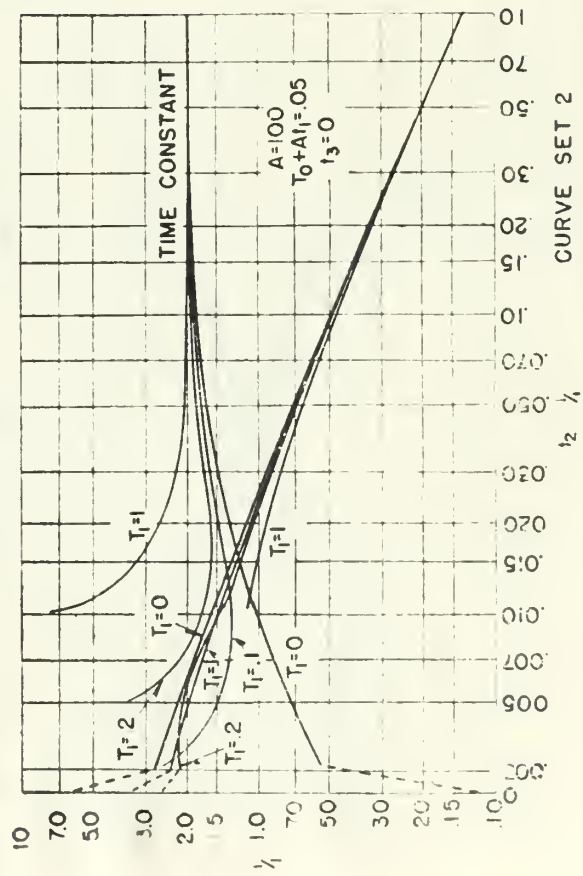
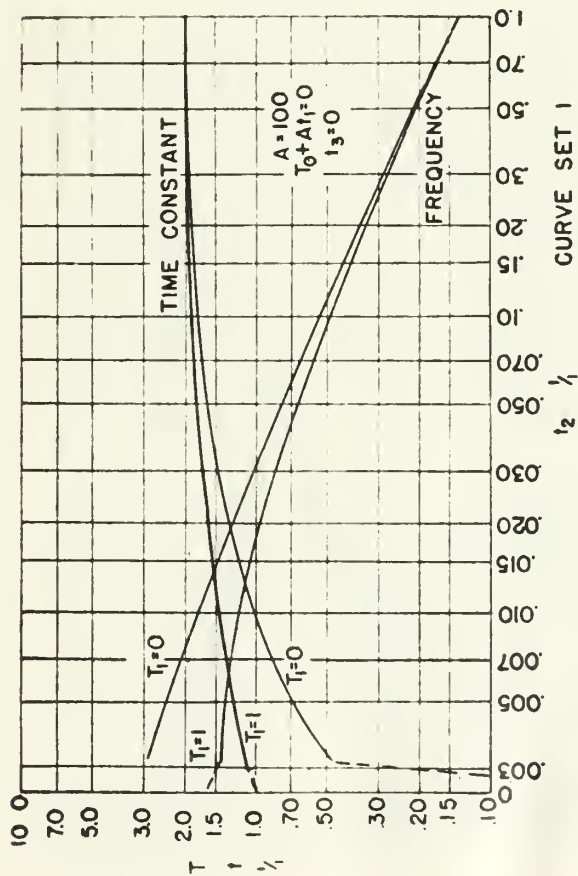


Figure 6 : SYSTEM CHARACTERISTICS



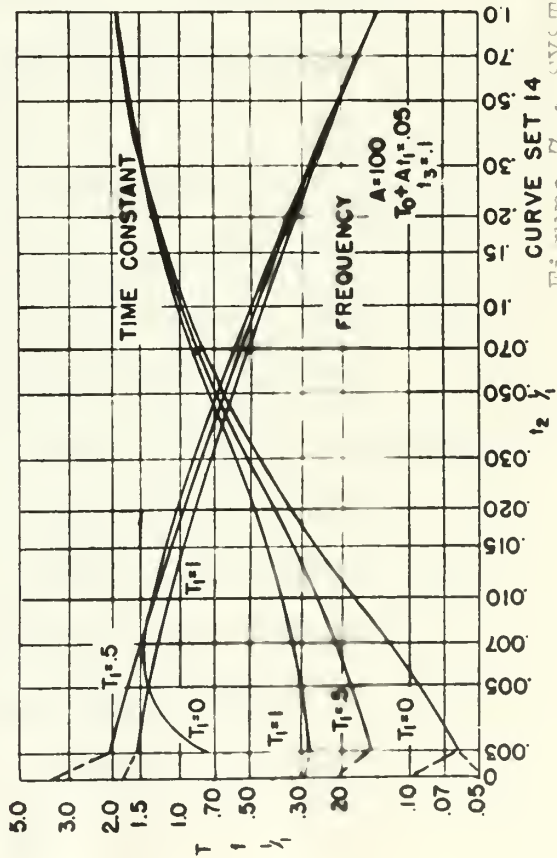
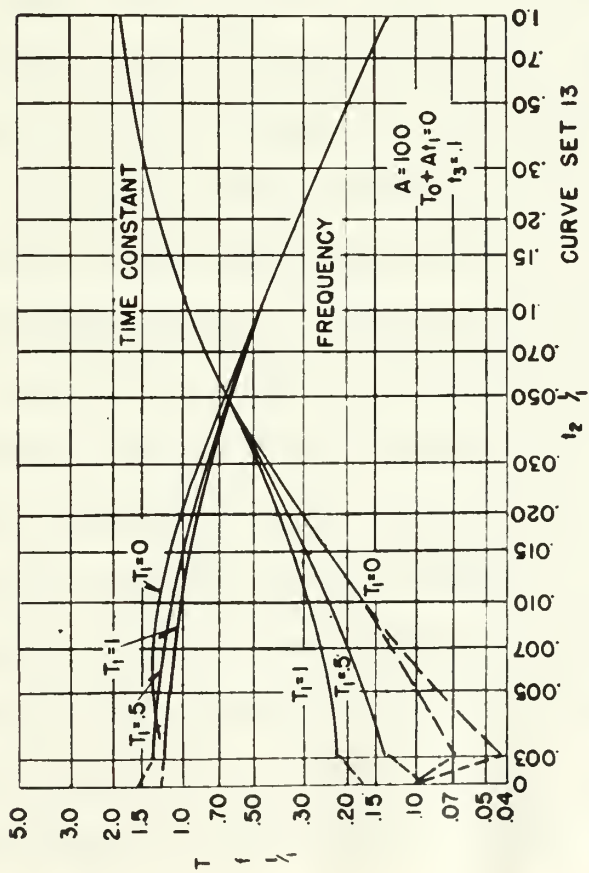
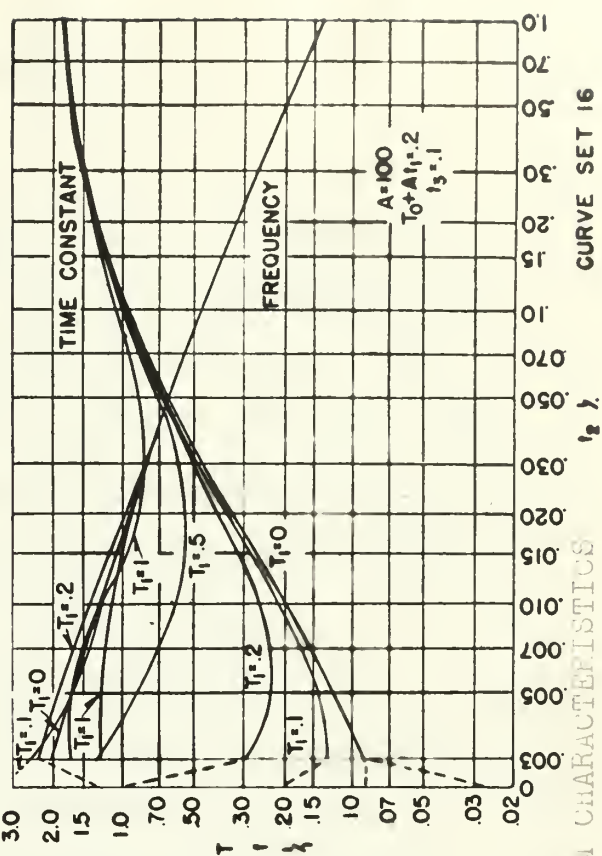
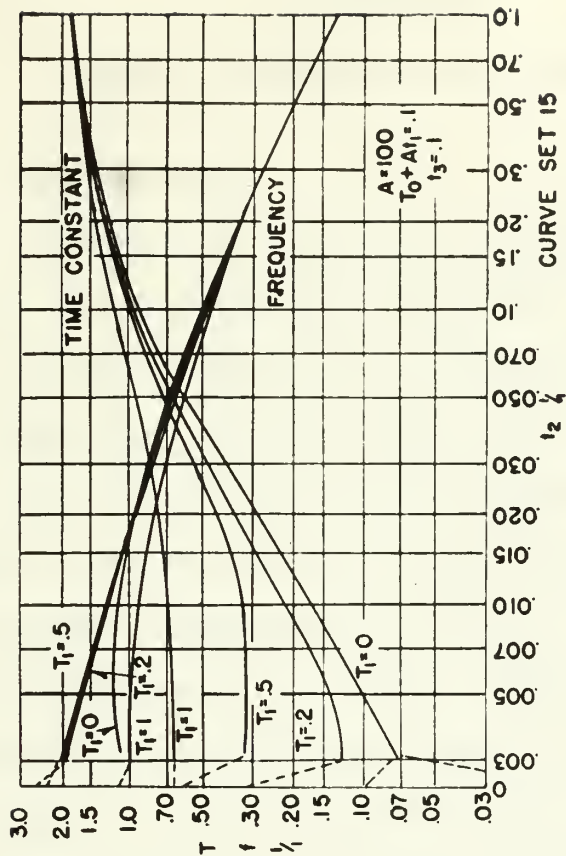


Figure 7 : SYSTEM CHARACTERISTICS



## CHAPTER II

### MAGNETIC AMPLIFIER CHARACTERISTICS

#### 1. Time constant.

The magnetic amplifier, although capable of high power gains, is nevertheless relatively slow to respond when used with rather conventional "garden variety" circuitry. This study will be concerned with "garden variety" circuitry such as would be available to the engineer who was attempting to design a control circuit using a magnetic amplifier. After talking with engineers at the U. S. Naval Research Laboratories and U. S. Naval Ordnance Laboratories it becomes apparent that new ideas in circuitry must be used to combat the slow response time of magnetic amplifiers. Much work along these lines is being done and the results of these studies should evidence themselves in a more widespread use of magnetic amplifiers.

Since each time constant in the regulation circuit is important, the time constant of the magnetic amplifier will be discussed here. It has been generally accepted that for all practical purposes a direct proportionality exists between a change in the initial flux and a change in average output voltage of the self-saturated magnetic amplifier. Since the saturation flux density is fixed, the average flux changes by half of any change of initial flux. Changes of average flux, initial flux and average load voltage are all proportional to each other and hence appear rather indiscriminately in response time formulas.





It is the average flux that, linking the control windings, opposes the change of average control current and determines the time constant of the magnetic amplifier. The problem of determination of the time constant is one of determining the average flux linkages per core per ampere of signal current which gives the inductance of the control winding. For clarity a schematic of a simple magnetic amplifier is sketched in Figure 8.





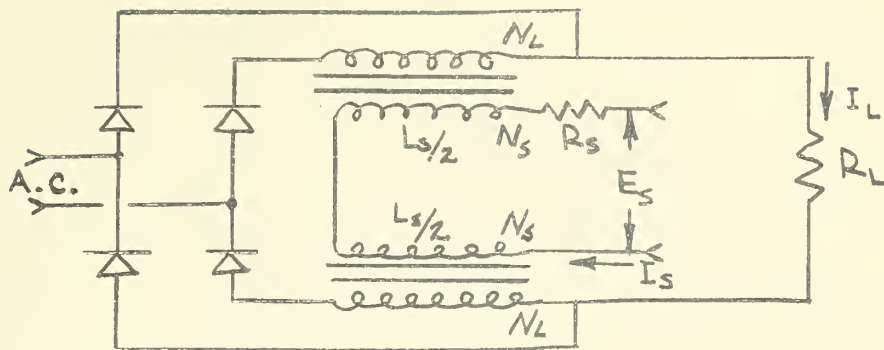


FIGURE 8  
MAGNETIC AMPLIFIER

Total control circuit inductance =  $L_s$

Total control circuit resistance =  $R_s$

$$T = L_s / R_s$$

$$L_s = \frac{2 \Delta \Phi_{ave} N_s}{\Delta I_s \cdot 10^8}$$

$$T = \frac{2 \Delta \Phi_{ave} N_s}{10^8 \Delta I_s R_s} = \frac{2 \Delta \Phi N_s}{10^8 \Delta E_s}$$

For the idealized case of sharp saturation, changes in average flux and output voltage are proportional.

$$e = \frac{N_L}{10^8} \frac{d\phi}{dt}$$

$$\Delta E_L = \frac{2 N_L \Delta \Phi \cdot 2f}{10^8}$$

$$\text{Substituting } T = \frac{\Delta E_L N_s}{2f N_L} = \frac{A_v}{2f}$$

where  $A_v$  = average gain on a common turns basis.



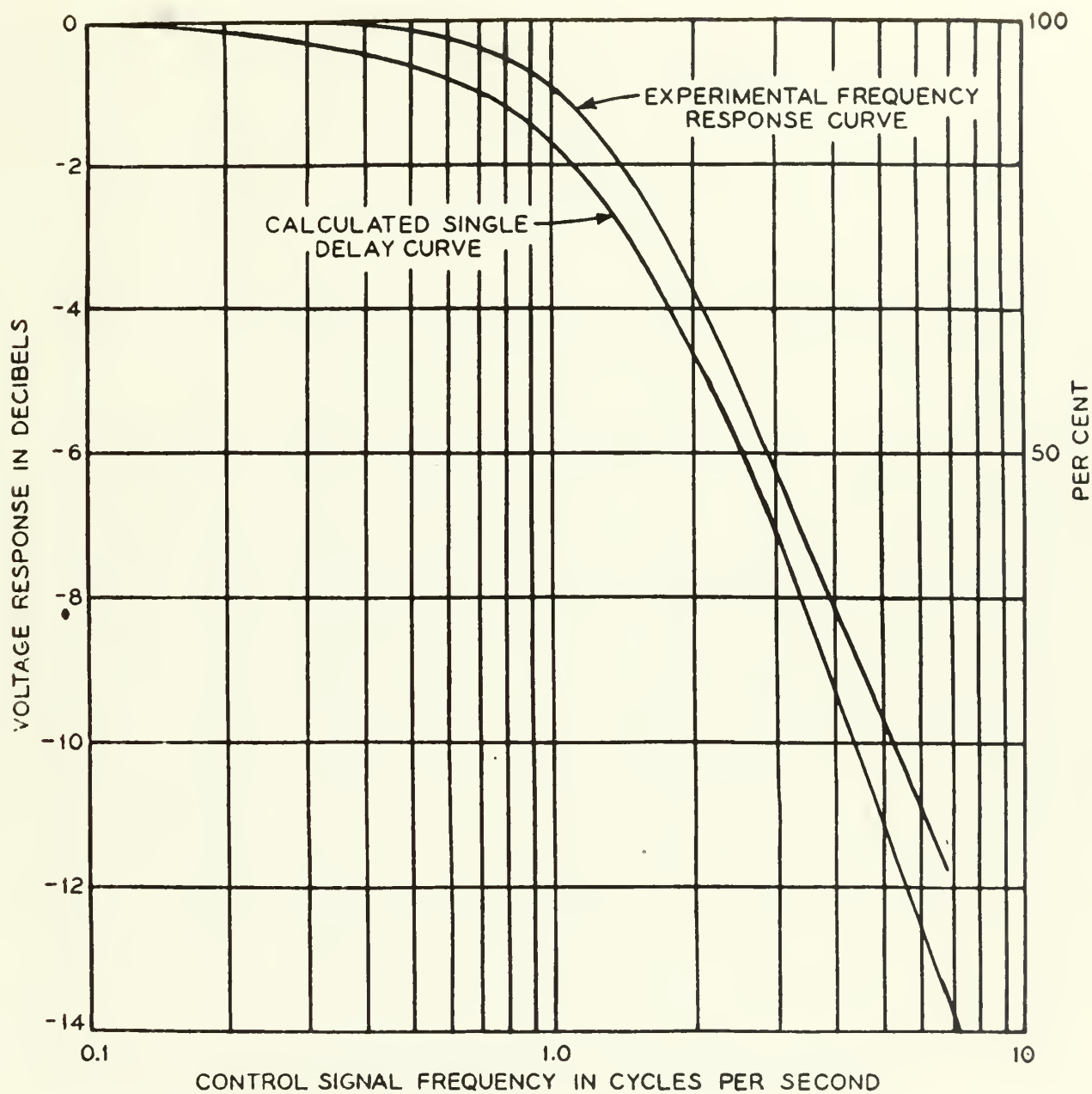


Figure 9  
MAGNETIC AMPLIFIER FREQUENCY RESPONSE



The above result gives a low value when the saturation curve deviates from a flat top, since it is based on a change in average flux which is only half the flux swing. The above proof that the magnetic amplifier can be represented by a single time constant may be tested by comparing its frequency response with that of a theoretical single delay system. Such a comparison is given in Figure 9, (4). The test amplifier follows the theoretical curve reasonably well, showing that the magnetic amplifier can justifiably be represented by a single time delay.

## 2. Amplification - time constant relation.

Since the time constant is determined by the flux linkages per unit signal voltage, changing the signal circuit resistance changes the power amplification and the time constant proportionally. This proportionality between the power gain and time constant as the signal circuit resistance is varied is illustrated by some results of a paper from N. O. L. by Mr. Edgar V. Wier (7) (see Figure 11). If power amplification is represented by  $A$ , the  $\frac{A}{T}$  ratio could be used as a figure of merit for comparison of amplifiers of same size and type operating in the same region. It would show up in a single figure the beneficial effects of high permeability iron, low-leakage rectifiers, and quality of design in respect to space utilization in lowering internal resistance, leakage resistance and so forth. Note that in Figure 11 the ratio deviates from a straight line in the low time constant region. This must be borne in mind

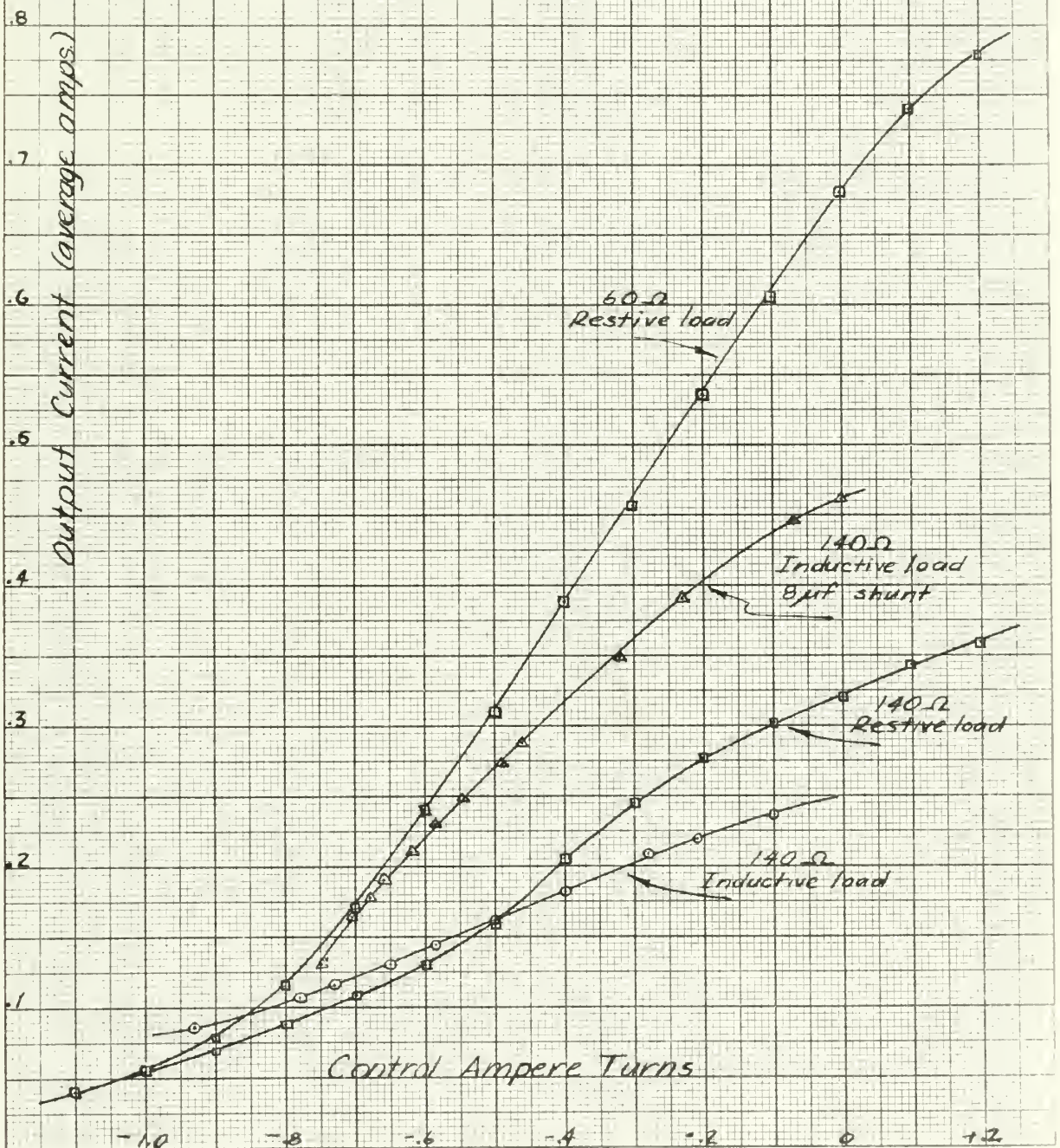


when working with magnetic amplifiers of normal circuitry in the low time constant region. This discrepancy exists because the original assumptions were based on a long-transient theory in which averages can be used extensively. As the time constant approaches one cycle of the supply frequency, the ratio drops off quite rapidly when the long-transient theory applies less accurately and the lamination and a.c. winding delays become a factor. Not shown in this figure are the differences in increasing and decreasing transients. If the back voltage induced in the a.c. windings by the applied signal voltage exceed the back voltage for one diode, a circulating current can flow for decreasing transients, thereby slowing the response. This does not occur for increasing transients since the induced voltages are counter to the diode conducting directions. Therefore, the figure of merit ratio will probably not receive industry standardization until its limitations have been more thoroughly explored. However, it still remains a most useful tool when comparing amplifiers for use in the design of a system.





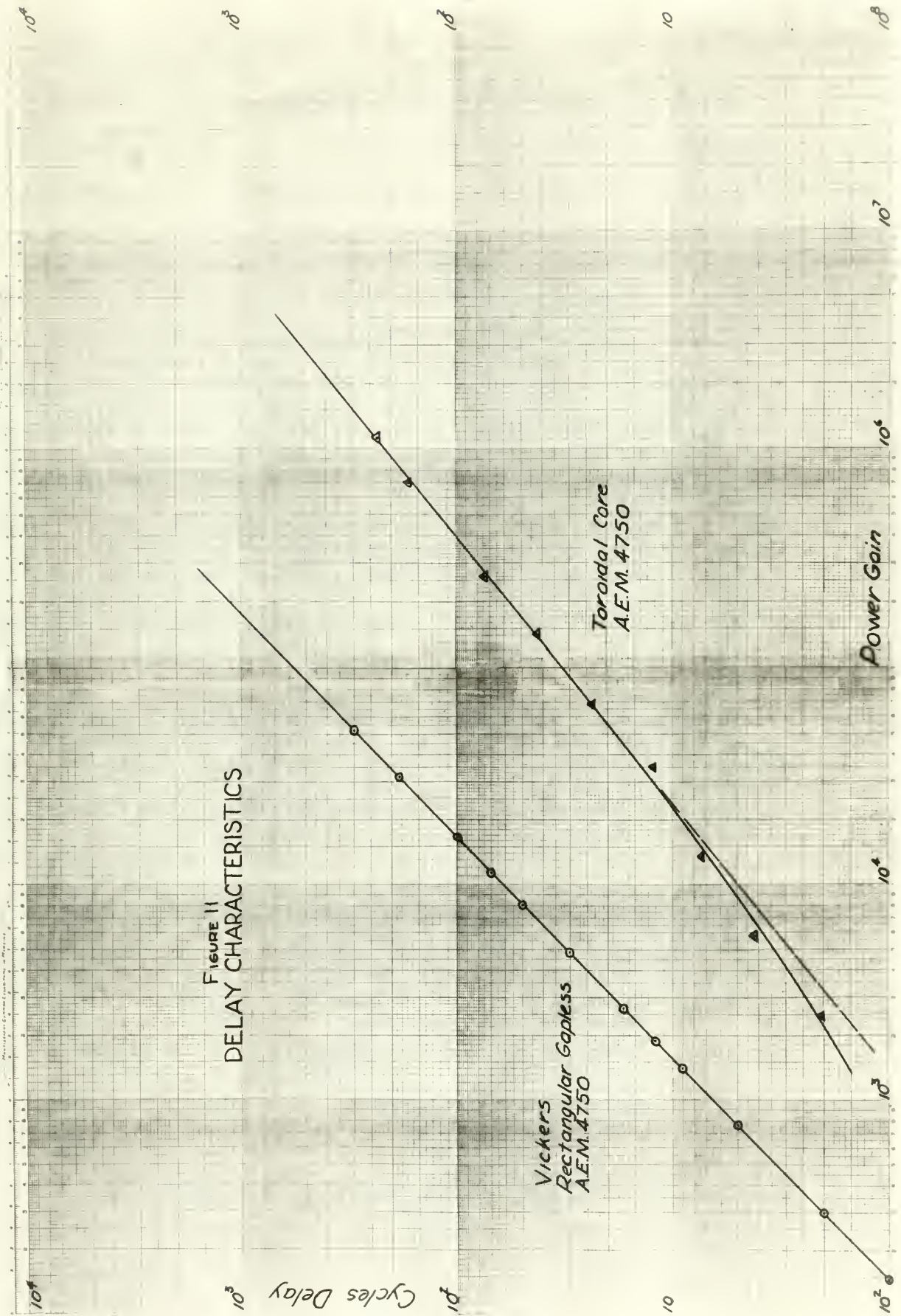
FIGURE 10  
MAGNETIC AMPLIFIER  
CONTROL CHARACTERISTICS  
VICKERS No. 2215



KEUFFEL & ESSER CO., N. Y. NO. 359H 12  
10  $\times$  10 to the  $\frac{1}{2}$  inch, 5th lines accented  
MADE IN U. S. A.









### 3. Inductive load.

In matching the magnetic amplifier to the load or vice versa it must be kept in mind that the load current from a magnetic amplifier is not pure d.c. For a highly inductive load the impedance "seen" by the magnetic amplifier is more than the d.c. resistance. This is shown from the current transfer curves for the Vickers magnetic amplifier #2215, Figure 10. The inductive load must be shunted by a rectifier oriented against the direction of current flow. This reduces the effect of the inductance. Figure 17 is an oscillograph of the output current of the magnetic amplifier. The current is by no means pure d.c. It is not difficult to see that the load impedance for such an output current will be much greater for an inductive load than for pure resistance. Comparison of curves in Figure 10 will illustrate this point. The inductive load was shunted by a rectifier oriented against the direction of current flow.

To smooth out this "d.c." output an eight microfarad condenser was put across the load. Figure 18 will illustrate how the ripple was reduced by this step. Transfer characteristics for the magnetic amplifier loaded with an inductive load shunted by a smoothing capacitor are shown in Figure 10.

It is interesting to note the effect of a large ripple in the field current of an alternator. Figure 13 is a composite of the various regulating currents demanded by



the alternator as the load is changed from zero to full load. The un-smoothed amplifier output current required to regulate is much less than either the "smoothed" amplifier output or the straight d.c. Since it is difficult to get something for nothing, an explanation is necessary. The "current" required was read by a d.c. meter which gives average current. The power from the un-smoothed amplifier current is of course greater than that of the d.c. or "smoothed" current. Also the average flux for a varying d.c. field current will be greater than the flux that would be picked off a saturation curve for the average current. This is due to the hysteresis loop effect. As can be seen from Figure 13 the "smoothed" amplifier current is very close to pure d.c. in its effect of producing flux in the alternator field.

A schedule for the design of a voltage regulating system will now be outlined, endeavoring to take into account the general regulation characteristics as well as those of the magnetic amplifier.





# CHAPTER III

## REGULATION SYSTEM

### 1. Design schedule.

#### Voltage Regulation System Design

#### Machine Characteristics:

- |                            |            |
|----------------------------|------------|
| 1. Number of phases        |            |
| 2. Cycles                  |            |
| 3. Terminal voltage        | $E_L$      |
| 4. No load field current   | $I_{fmin}$ |
| 5. Full load field current | $I_{fmax}$ |
| 6. Field Resistance        | $R_f$      |

#### Assumptions:

- |                               |              |
|-------------------------------|--------------|
| 7. Max voltage change         | $\Delta E_L$ |
| 8. Max control voltage change | $\Delta E_c$ |

#### Magnetic Amplifier:

- |  |                                       |
|--|---------------------------------------|
| 9. Regulation power  | $P_R = (I_{fmax} - I_{fmin})^2 R_f$   |
| 10. Volume of core   | $V_c = \frac{P_R}{W_c}$               |
| 11. Change in control current<br>(from transfer characteristics) | $\Delta I_c$                          |
| 12. Control circuit resistance                                   | $R_s = \frac{\Delta E_c}{\Delta I_c}$ |
| 13. Change in field voltage                                      | $\Delta E_f = \Delta I_f R_f$         |
| 14. Voltage gain   | $A_v = \frac{\Delta E_f}{\Delta E_c}$ |
| 15. Power gain   | $A = \frac{(A_v)^2 R_s}{R}$           |



16. From Amplification vs. Time Constant Curve:

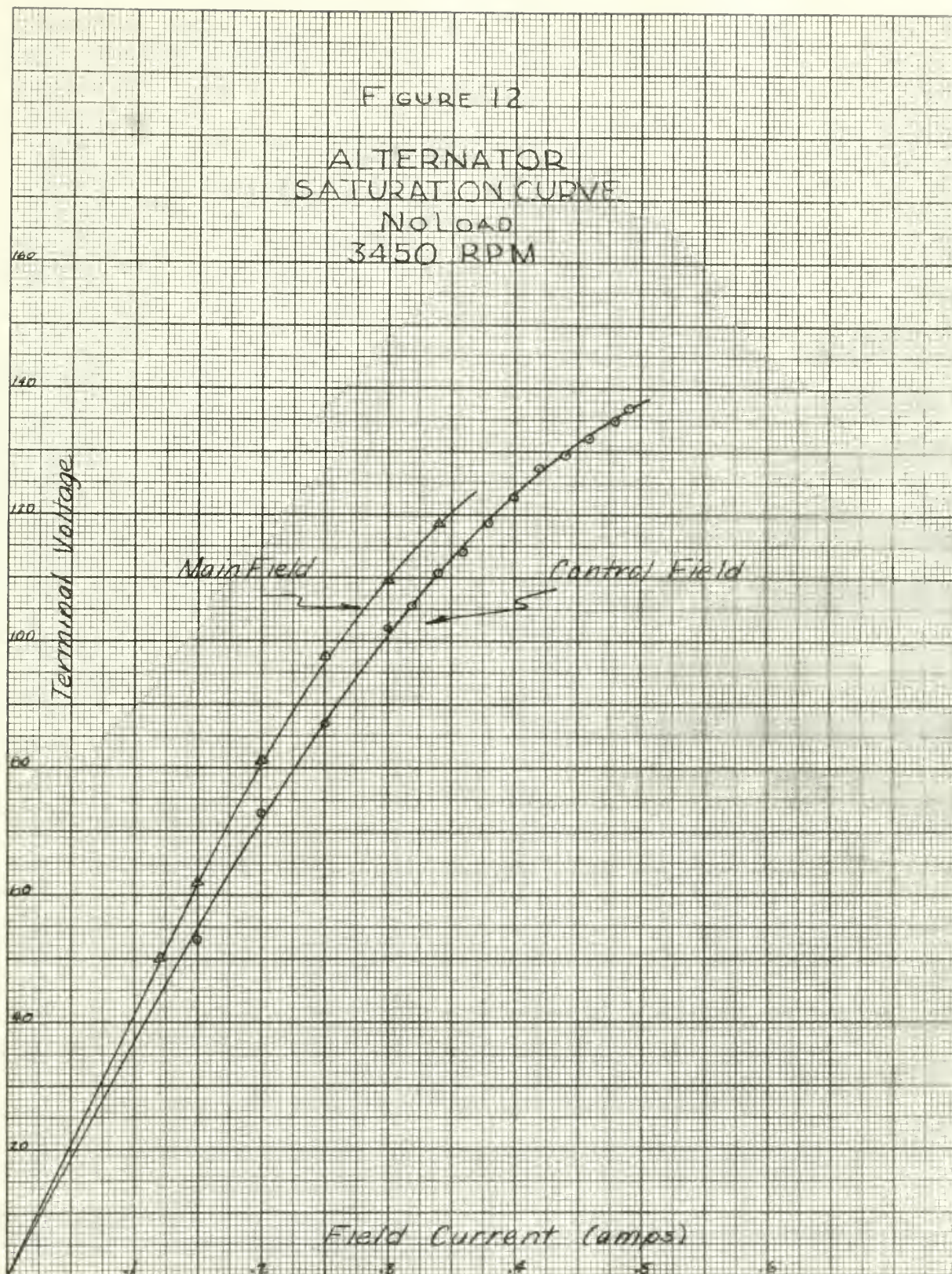
Time Constant of Amplifier = T

2. A system designed.

In order to further amplify and possibly clear up any ambiguous points which may exist in the design schedule, two designs will be carried out. First, a system to regulate a 2KVA, 400 cycle, single phase, 110 volt alternator will be designed. This alternator is constant speed, constant frequency, twin field originally designed for the exciter to feed one field while an electronic regulator controls the other field for regulation purposes, well suited for this problem except for the size of the field resistance. Figure 15 is a schematic of the circuit.

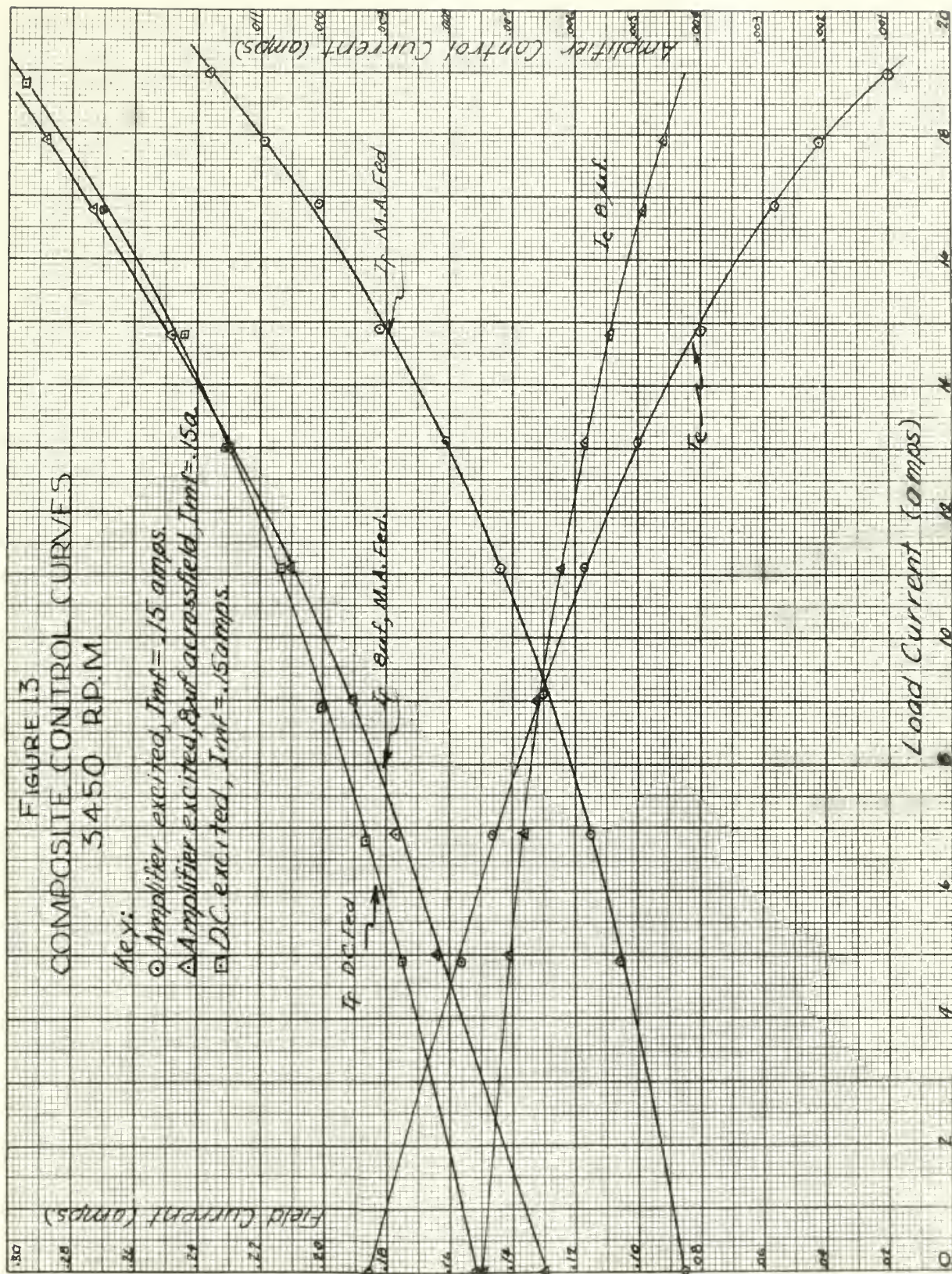






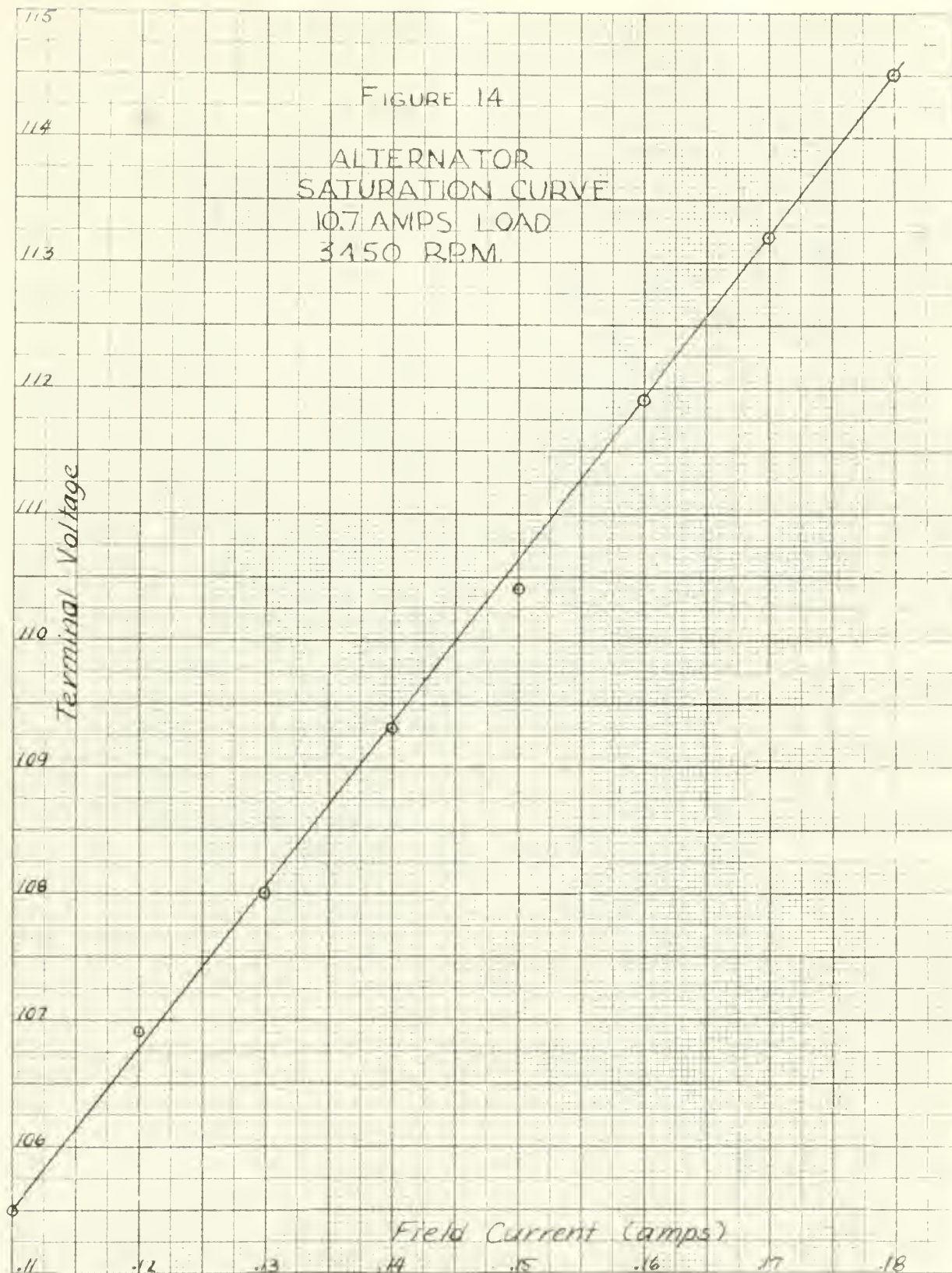














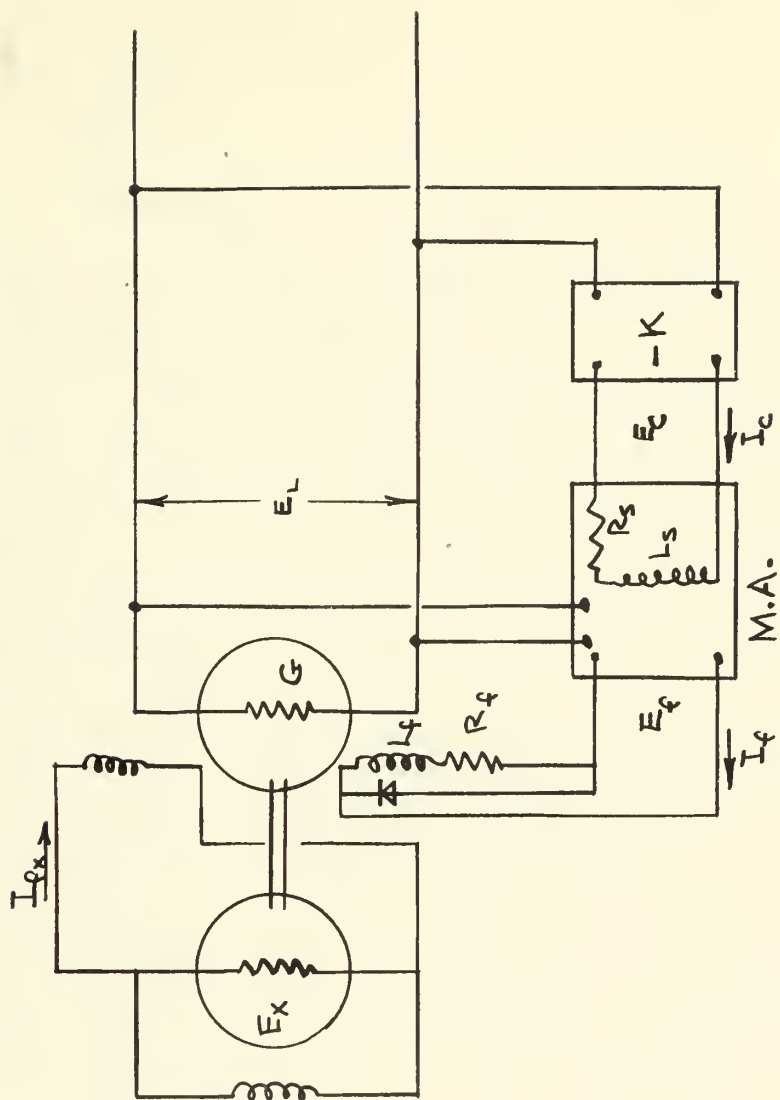


FIGURE 15  
SCHEMATIC-REGULATION CIRCUIT



## Voltage Regulation System Design

### Machine Characteristics:

1. Single phase
2. 400 cycles
3.  $E_L = 110$  volts
4.  $I_{fmin} = .085$  amps (for  $I_{fx} = .15$  amps)  
obtained from Figure 13.
5.  $I_{fmax} = .224$  amps. (for  $I_{fx} = .15$  amps)
6.  $R_f = 140$  ohms

### Assumptions:

7.  $\Delta E_L = 1$  volt
8.  $\Delta E_c = 5$  volts

Note: This  $\Delta E_c$  is a pure assumption since the voltage sensing circuit is not considered here.

### Magnetic Amplifier:

$$9. P_R = (\Delta I_f)^2 R_f = (.139)^2 (140) = 2.7 \text{ watts}$$

$$10. V_c = \frac{P_R}{W_c} = \frac{2.7}{41} = .066 \text{ in}^3$$

where  $W_c$  is watts per volume of core in inches.

Note: This volume was too small for any of the Vickers magnetic amplifiers. Since Vickers was assumed to be representative, and was to be the source of the magnetic amplifier to be used in the actual laboratory mock-up, Vickers #2215 was chosen so as to be of general use in the laboratory. When choosing the magnetic amplifier one must ascertain the load resistance at which the amplifier



delivers full output power. This load resistance must be made compatible with the resistance of the actual load. Since it was necessary to order a larger (more powerful) amplifier than necessary, matching the load resistance to the optimum load resistance of the amplifier was passed up in favor of current output matching. For a 140 ohm load the current output for the #2215 is just about correct for the current demand. Although the amplifier is not being used to its fullest extent powerwise, it is being fairly well extended over its current output range by the 140 ohm load. In this respect the design was deviated from the ideal.

$$11. \Delta I_c = .0074 \text{ amps}$$

From Figure 10, using 100 turn control winding

$$12. R_s = \frac{\Delta E_c}{\Delta I_c} = \frac{.5}{.0074} = 67.6 \text{ ohms}$$

$$13. \Delta E_f = \Delta I_f R_f = (.139)(140) = 19.45$$

$$14. A_v = \frac{\Delta E_f}{\Delta E_c} = \frac{19.45}{.5} = 38.9$$

$$15. A = \frac{(A_v)^2 R_s}{R} = \frac{(38.9)^2 67.6}{140} = 732$$

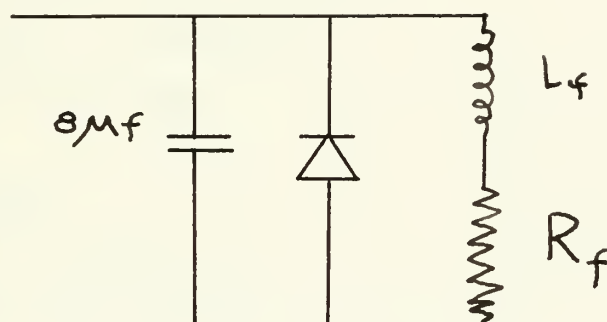
$$16. T = 4.8 \text{ cycles} = .012 \text{ seconds}$$

Note: This is from Figure 11, and is for 63% steady state.





For a comparison the design will be worked out for the arrangement in which the field is shunted by an 8 micro farad condensor. The circuit is the same as Figure 15 on page 33 with the exception shown below.





## Voltage Regulation System Design

### Machine Characteristics:

1. Single phase
2. 400 cycles
3.  $E_L = 110$  volts
4.  $I_{fmin} = .130$  amps (for  $I_{fx} = .15$  amps)

Obtained from Figure 13.

5.  $I_{fmax} = .296$  amps
6.  $R_f = 140$  ohms

### Assumptions:

7.  $E_L = 1$  volt
8.  $E_c = .5$  volts

### Magnetic Amplifier:

9.  $P_R = (\Delta I_f)^2 R_f = (.166)^2 (140) = 3.86$  watts
10.  $V_c = \frac{3.86}{41} = .0942$  in<sup>3</sup>

Using Vickers #2215:

11.  $\Delta I_c = .0031$

From Figure 10, using 100 turn control winding

12.  $R_s = \frac{.5}{.0031} = 161.2$  ohms
13.  $\Delta E_f = (.166)(140) = 23.2$
14.  $A_v = \frac{23.2}{.5} = 46.4$
15.  $A = \frac{(46.4)^2 (161.2)}{(140)} = 2480$
16.  $T = 15.5$  cycles  $= .039$  seconds



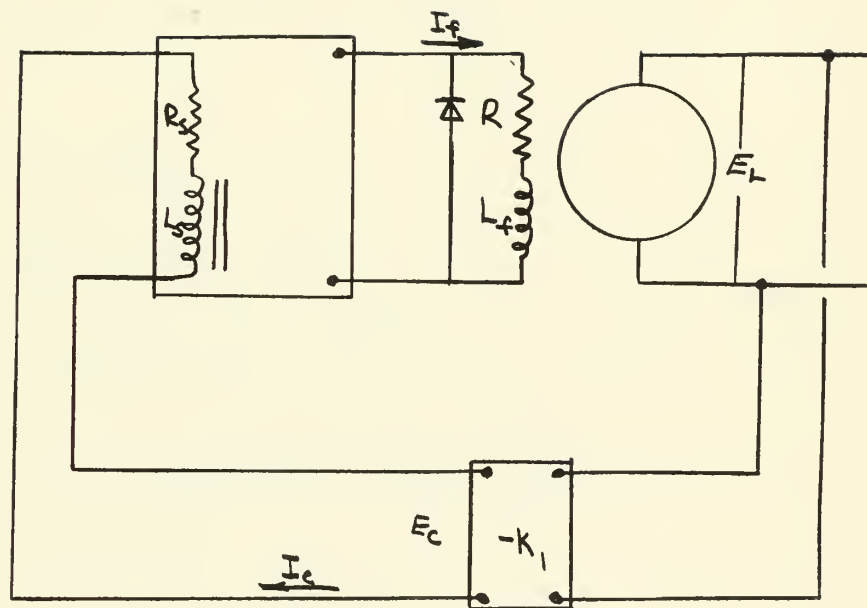


FIGURE 16  
REGULATION CIRCUIT



### 3. Analytical development.

For comparison the regulation problem just presented will be developed and solved analytically. The schematic is shown in Figure 16. The same assumptions and procedure will be used as was used previously in the general developments. All voltages and currents are changes of voltage and current.

$$\Delta \overline{E}_c = -K_1 \Delta \overline{E}_L$$

$$\Delta \overline{E}_L = \Delta \overline{I}_c (R + L_s S)$$

$$\Delta \overline{I}_f = k (\Delta \overline{I}_c)$$

where  $k$  = slope of current transfer curve.

$$\Delta \overline{E}_f = k \Delta \overline{I}_c R_f$$

$$\therefore \Delta \overline{E}_f = \frac{\Delta \overline{E}_c k R_f}{L_s \left( \frac{R}{L_s} + S \right)} = \frac{\Delta \overline{E}_c k R_f}{L_s \left( \frac{R}{L_s} + S \right)}$$

$$\text{where } \frac{L_s}{R_s} = T_1$$

$$\Delta \overline{I}_f = \frac{\Delta \overline{E}_f}{R_f + L_f S} = \frac{\Delta \overline{E}_f}{L_f \left( \frac{1}{T_2} + S \right)}$$

$$\text{where } \frac{L_f}{R_f} = T_2$$





When a voltage disturbance ( $C u(t)$ ) is imposed on the system:

$$\Delta \overline{E_c} = -K_1 (\Delta \overline{E_L} - \overline{C u(t)})$$

$$\Delta \overline{I_f} = \frac{\Delta \overline{E_c} k R_f}{L_s L_f (s + \frac{1}{T_1})(s + \frac{1}{T_2})}$$

$$\Delta \overline{E_L} = K_G \Delta \overline{I_f}$$

$$\Delta \overline{E_L} = \frac{\Delta \overline{E_c} k K_G R_f}{L_f L_s (s + \frac{1}{T_1})(s + \frac{1}{T_2})}$$

$$\Delta \overline{E_L} = \frac{K_1 k K_G R_f \overline{C u(t)}}{L_f L_s (s + \frac{1}{T_1})(s + \frac{1}{T_2}) + K_1 k K_G R_f}$$

$$\text{Let } M = \frac{K_1 k K_G R_f}{L_f L_s}$$

$$\Delta \overline{E_L} = \frac{M \overline{C u(t)}}{(s + \frac{1}{T_1})(s + \frac{1}{T_2}) + M}$$

$$\Delta \overline{E_L} = \frac{MC}{1 + M} \quad \text{— exponentially decaying transients or oscillations.}$$

Using measured values for the various constants, the regulation problem will be solved analytically for a load of approximately 10 amperes.



Constants	Source
$T_1 = .012$ seconds	design data
$1/T_1 = 83.4$	
$R_s = 67.6$ ohms	design data
$k = 18.8$	Figure 10
$L_s = .812$ h	$L_s = R_s T_1$
$R_f = 140$ ohms	measured
$L_f = .56$ h	measured
$T_2 = .004$ seconds	$T_2 = L_f / R_f$
$1/T_2 = 250$	
$K_1 = .5$	assumed
$K_G = 128$	Figure 14

From previous development, page 41:

$$\Delta \bar{E}_L = \frac{M \overline{Cu(t)}}{(s + \frac{1}{T_1})(s + \frac{1}{T_2}) + M}$$

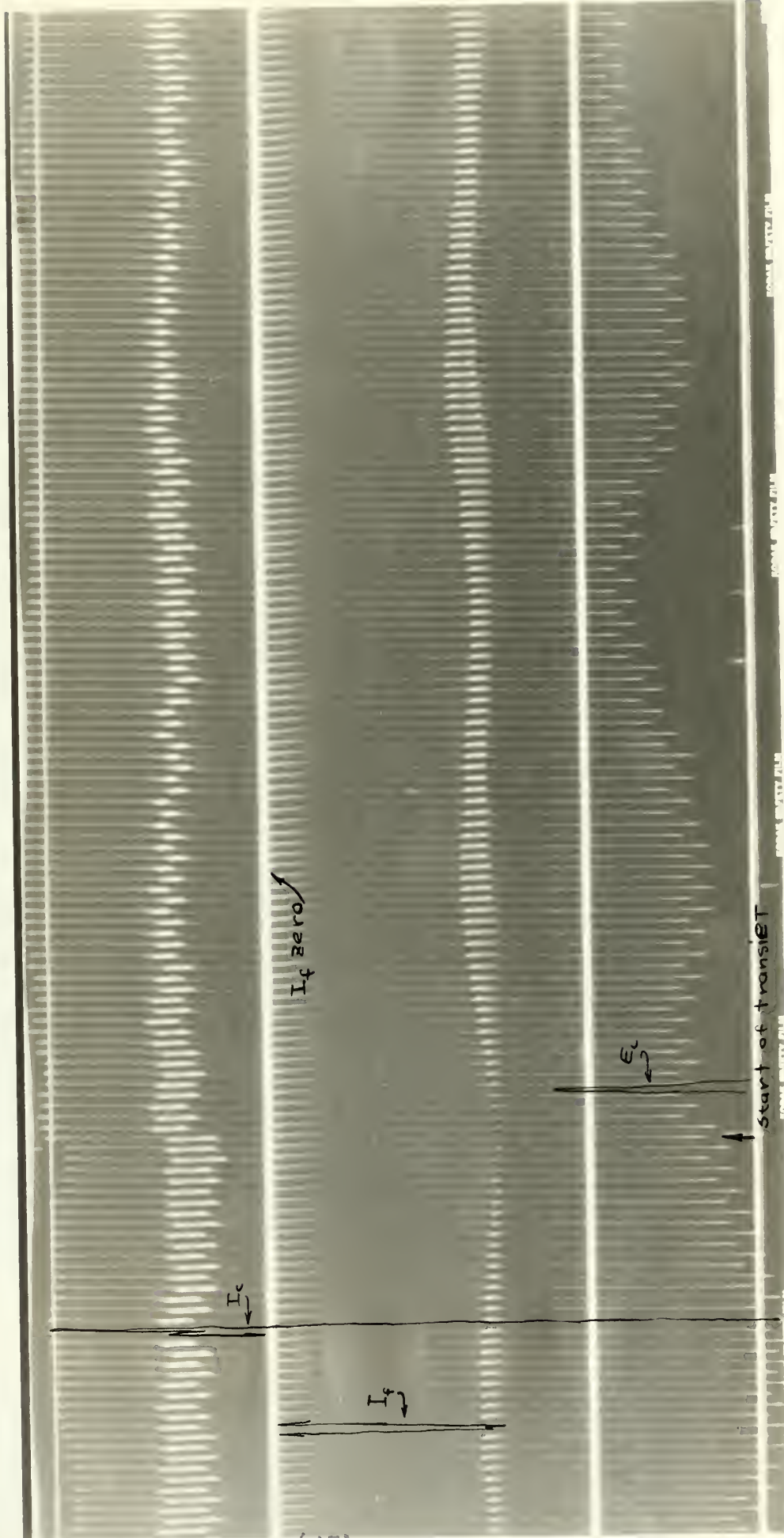
$$\text{where } M = \frac{K_1 k K_G R_f}{L_f L_s} = \frac{18.8(128)(140)}{2(.812)(.56)} = 371,000$$

$$\Delta \bar{E}_L = \frac{MC}{s[(s + 83.4)(s + 250) + 371,000]} = \frac{MC}{s[s^2 + 333.4s + 391,900]}$$

$$\Delta \bar{E}_L = \frac{MC}{s(s + \alpha - j\beta)(s + \alpha + j\beta)}$$

$$\text{where } \alpha = 166.7 \quad \beta = 604$$





(43)

(44)

Figure 17

OSCILLOGRAPH No. 1



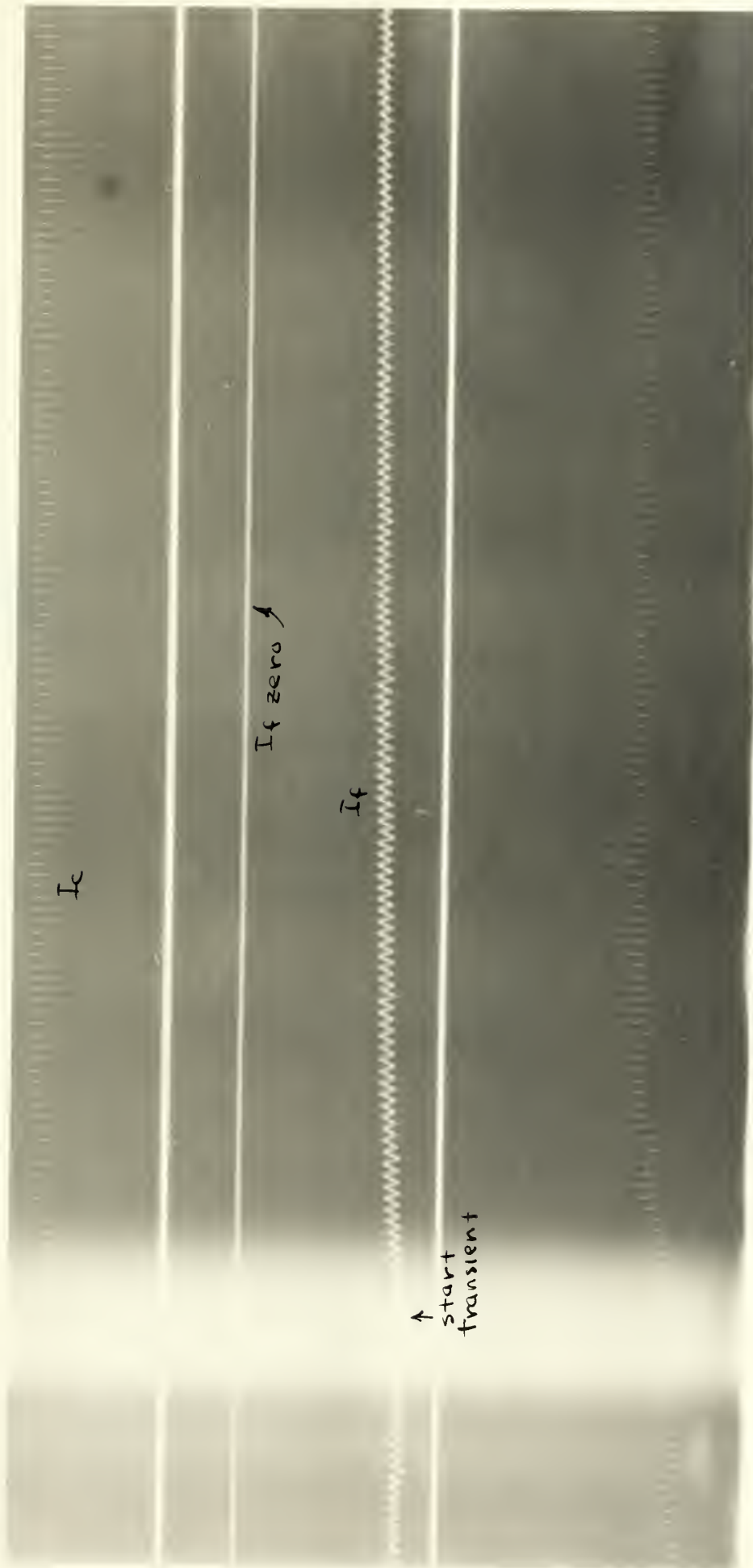


Figure 18  
OSCILLOGRAPH No. 2





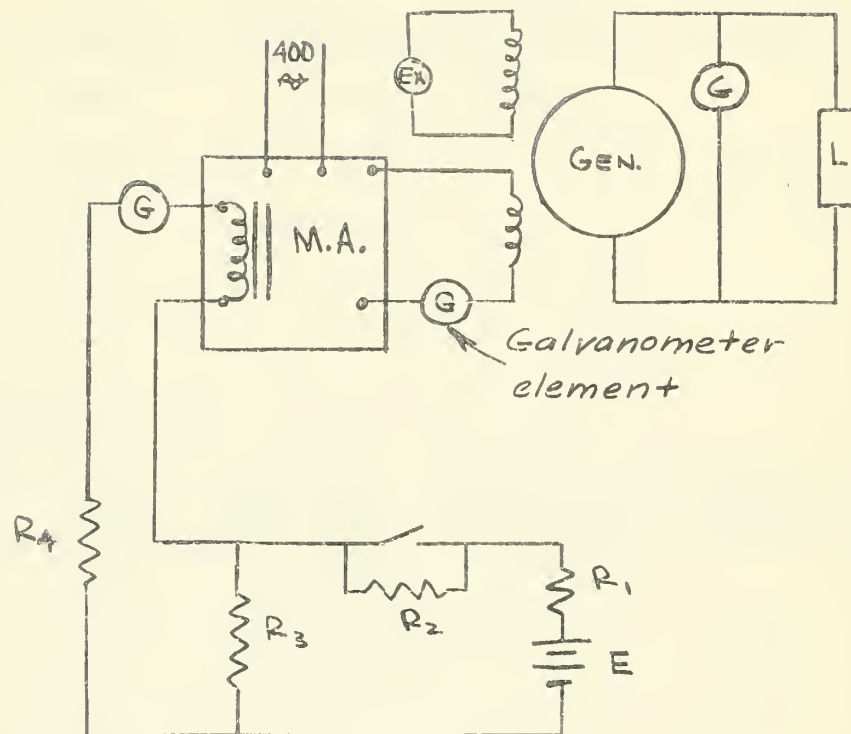


FIGURE 19  
OSCILLOGRAPH RECORDING  
CIRCUIT



TABLE 1  
Oscillograph Data

1. Oscillograph No. 1

	Initial value	Final value	Observed Time cnst.	Predicted Time cnst.	Observed change	Predicted change
$I_c$	.005	.0071	5.87 cs.	4.8 cs.		
$I_f$	.132	.112	15.1 cs.		.020	.041
$V_L$	108	105.2	24.0 cs.		5.45	2.80
$I_L$	9.7	9.5				

2. Oscillograph No. 2

	Initial value	Final value	Observed Time cnst.	Predicted Time cnst.	Observed change	Predicted change
$I_c$	.00505	.0074	5.42 cs.	15.5 cs.		
$I_f$	.269	.135	17.2 cs.		.134	.128
$V_L$	115.9	100.0			15.9	16.3
$I_L$	11.6	9.9				



#### 4. Oscillograph measurements.

In order to check the system designed by the design schedule, a step voltage was put into the control circuit of the magnetic amplifier and pictures taken with an oscillograph of the various transients involved. Since the circuit is not closed, an independent source of 110 V, 400 cycle A.C. was obtained to supply the magnetic amplifier. The test alternator was connected to a load and fed by the magnetic amplifier. Figure 19 will show this set up. Care was taken to insert a step voltage in the control circuit of the magnetic amplifier without changing the resistance of the control circuit. This was done by making  $R_1$   $R_3$  and  $R_2$   $R_3$ . The load was set for approximately 10 amperes. Table 1 is a tabulation of the results of the oscillographs. Figure 17 is the oscillograph of the transient in the system without an 8 microfarad condenser across the field of the alternator. In order to distinguish the peculiar wave shapes one of each of the  $I_f$  and  $I_c$  current waves is inked in. Notice that the output of the magnetic amplifier is not d.c. In fact the output current drops to zero once each cycle (at 800 c.p.s.). The control current also has a.c. superimposed upon the d.c. The peak to peak value is approximately 40 m.a. This is induced by the output windings on the cores. The control windings should be wound so as to cancel the induced currents, but obviously they are not exactly balanced. Rough measurements and calculations indicate that the balance is about 4% off. At a lesser gain the added resistance in the control circuit would reduce this effect.



Table 1 gives initial and final values as read on the meters in the circuit. Also included are the time constants, as measured from the oscillographs, of the magnetic amplifier ( $I_e$ ) and the circuit as a whole ( $I_f$ ). The predicted time constants are also included; these were taken from previous calculations on pages 35 and 37. Note that the time constant for the magnetic amplifier in Figure 17 (5.87) was very close to the predicted time constant of 4.8 cycles. The time constant of the amplifier in Figure 18 was measured to be 5.42 cycles which is considerably less than the predicted 15.5 cycles.

Figure 18 illustrates the smoothing effect of the condenser across the inductive load. Note the difference in ripple between the two field currents of Figures 10 and 13, the 8 microfarad condenser permits the magnetic amplifier to operate as though the load is purely resistive and of smaller resistance. The inductive load with the aid of the by-pass rectifier but without the by-pass condenser restricts the amplifier as a larger load resistance would restrict it. Evidently it is beneficial to use by-pass condensers when driving inductive loads.

## 5. Summary

To sum up, this thesis has explored the basic facts concerned in a regulation circuit. It has attempted to point out some of the operating characteristics of the magnetic amplifier using "garden variety" circuits. It has put forth a design schedule utilizing the relations developed and characteristics discussed. It has attempted, in a limited





sense to prove the basic tenets assumed by recording on film the transient produced by a step voltage in the control circuit. It is unfortunate that further recordings could not be made, due to power failure, to resolve the differences that did exist between recorded and analytical results. However, it is felt that the answer lies in added circuitry. This writer feels that a thorough understanding of the regulation problem would permit rapid design of regulating systems, and easy adaptation of new phenomena to a regulation system.



## BIBLIOGRAPHY

1. Evans, W. R. Control system synthesis by root locus method. American Institute of Electrical Engineers, Transactions. Volume 69, part I pages 66-69.
2. Finzi, L. A. and D. C. Beaumariage. General characteristics of magnetic amplifiers. American Institute of Electrical Engineers, Transactions. Volume 69, part II pages 919-927.
3. Harder, E. L. Solution of the general voltage regulator problem by electrical analogy. American Institute of Electrical Engineers, Transactions. Volume 66, pages 815-824.
4. Harder, E. L. and W. F. Horton. Response time of magnetic amplifiers. American Institute of Electrical Engineers Transactions. Volume 69, part II, pages 1130-1138.
5. Ruedenberg, R. Transient performance of electrical power systems. New York, McGraw-Hill. 1950
6. Smith, E. J. Determination of steady-state performance of self-saturating magnetic amplifiers. American Institute of Electrical Engineers, Transactions. Volume 69, part II, pages 1309-1317.
7. Weir, E. V. NOLM 10555. October 20, 1949.  
NOLM 10864. June 14, 1950.
8. .... Magnetic amplifier design handbook. Vickers Electric Division of Vickers Inc., St. Louis, Missouri.















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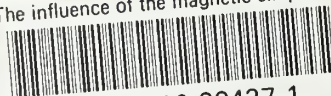
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